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181ENGO5/032  
Mechatronics

Date No.

Question 1:

$$T_{\text{in}} = 10^{\circ}\text{C}$$

$$T = 20^{\circ}\text{C} @ 5 \text{ mins}$$

$$T_{\text{ambient}} = 25^{\circ}\text{C}$$

$$\frac{dT}{dt} \propto (T - T_A)$$

$$\frac{dT}{dt} = k(T - T_A)$$

$$\frac{dT}{dt} = k(T - 25)$$

$$dT = k(T - 25) dt$$

$$\int \frac{dT}{T - 25} = \int k dt$$

$$\ln(T - 25) = kt + C$$

Integrating both sides

$$\ln(T - 25) = kt + C$$

$$T - 25 = e^{kt+C}$$

$$T - 25 = e^{kt} \cdot e^C$$

$$T - 25 = Ae^{kt}$$

$$T = Ae^{kt} + 25$$

at initial condition  $t=0, T=10^{\circ}\text{C}$

$$10 = Ae^0 + 25$$

$$A = -15$$

$$T = -15e^{kt} + 25$$

at  $T = 20^{\circ}\text{C}, t = 5 \text{ mins}$

$$20 = -15e^{5k} + 25$$

$$-5 = -15e^{5k}$$

$$e^{5k} = \frac{1}{3}$$

$$\ln e^{5k} = \ln \left( \frac{1}{3} \right)$$

$$5k = \ln \left( \frac{1}{3} \right)$$

$$k = \frac{\ln \left( \frac{1}{3} \right)}{5}$$

$$k = 0.231$$

$$k = 0.05$$

$$T = 35e^{0.05t} - 25$$

$$T = 24.9 \text{ at } t = ?$$

$$24.9 = 35e^{0.05t} - 25$$

$$49.9 = 35e^{0.05t}$$

$$e^{0.05t} = \frac{49.9}{35}$$

$$0.05t = \ln \left( \frac{49.9}{35} \right)$$

$$0.05t = 0.355$$

$$t = 7.1 \text{ minutes}$$

Mathcad - [Untitled2]

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$t \geq 0, 1..35$

$T(t) = -15 \exp(-0.21972 \cdot t) + 25$

T(t) =

24.138
24.308
24.444
24.554
24.642
24.713
24.769
24.815
24.851
24.881
24.904
24.923
24.938
24.95
24.96
...

Calculator

Graph

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Mechanics

Date

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Question 2

~~20~~  $T_0$  of environment is  $25^\circ\text{C}$

$$\frac{dT}{dt} = k(T - 25)$$

$$\int \frac{dT}{(T-25)} = \int k dt$$

$$\ln(T-25) = kt + c$$

$$T - 25 = e^{kt+c}$$

$$T - 25 = e^{kt} \cdot e^c$$

$$T - 25 = T_0 e^{kt}$$

$$T = T_0 e^{kt} + 25$$

At  $t = 0, T = 10^\circ\text{C}$

$$10 = T_0 e^{k \cdot 0} + 25$$

$$10 = T_0 + 25$$

$$10 - 25 = T_0$$

$$\therefore T_0 = -15^\circ\text{C}$$

$$\therefore T = -15e^{kt} + 25$$

but when  $t = 5 \text{ min}, T = 20^\circ\text{C}$

$$20 = -15e^{k \cdot 5} + 25$$

$$20 - 25 = -15e^{k \cdot 5}$$

$$\frac{-5}{-15} = e^{k \cdot 5}$$

$$0.33 = e^{k \cdot 5}$$

$$\ln 0.33 = \frac{k \cdot 5}{5}$$

$$k = -0.2197$$

$$\therefore T = -15e^{-0.2197t} + 25^\circ\text{C}$$

To find  $t$  at  $T = 24.9^\circ\text{C}$

$$24.9^\circ\text{C} = -15e^{-0.2197t} + 25^\circ\text{C}$$

$$24.9^\circ\text{C} - 25^\circ\text{C} = -15e^{-0.2197t}$$

$$-0.1 = -15e^{-0.2197t}$$

$$\frac{-0.1}{-15} = e^{-0.2197t}$$

$$\frac{0.667}{15} = e^{-0.2197t}$$

$$\ln 0.667 \times 10^{-3} = -0.2197t$$

$$\ln 6.67 \times 10^{-3} = -0.2197t$$

$$-5.010 = -0.2197t$$

$$\frac{-5.010}{-0.2197} = -0.2197t$$

$$-0.2197t = -0.2197t$$

$$t = 22.80 \text{ min}$$